Maths Calculation Policy

"By working together we learn, we achieve, we care"

Policy Consultation & Review

This policy is available on request from the school office.

This policy will be reviewed in full by the Governing Body on a two-yearly basis. This policy was last reviewed and agreed by the Governing Body on January 2023. It is due for review on January 2025.

Signature Headteacher Date:

Signature Chair of Governors Date:

Introduction

This policy has been written under the guidance of NCETM resources, Norfolk MathsHub and national researchers' guidance using current up to date information. Therefore, this policy ensures consistency and progression of the key procedures throughout the school and reflects a whole school agreement.

We place high expectations on how the children write the numbers used in simple number sentences. This is why we have decided to use books marked with squares on each page throughout the school, with larger squares used in Key Stage 1 compared to those used in Key Stage 2. However, the emphasis in this policy is of encouraging informal drawings and jottings to support the children's mathematical thinking. Whilst the actual number sentence should be recorded clearly, every opportunity will be given to children to use individual jottings and varied representations, which they may need to help them solve specific problems.

This is a crucial support and at Reedham School we recognise the importance of using informal jottings alongside concrete and pictorial representations, which help to clarify children's thinking of how the problem is to be solved. This develops confidence and a better understanding of the numbers being used and how they are manipulated within calculations – children will have conceptual understanding. When answering problems, the jottings used can be recorded near to a question or on the alternate page in their books for all Key Stage 1 and Key Stage 2 children. Even when children are securely working in stage 5 and moving towards more formalised approaches to written calculations we will still ask them to use jottings to support their thinking if they feel they need to.

The policy concentrates on the use of the empty number line as a jotting aid to mental calculation and on the introduction of more formal paper and pencil procedures. Children will be encouraged to look at a calculation or problem and then decide on the best method to use. It may be using pictures or drawings or a bar model, a mental calculation with or without jottings, a more structured method or the use of a calculator. Our long-term aim is for children to be able to select an efficient method of their choice, which is also appropriate for the given task.

Although the focus of this policy is on pencil and paper procedures, it is important to recognize that the ability to calculate mentally lies at the heart of the National Curriculum 2014. In every written method there is an element of mental processing. The mental methods in the statutory framework for teaching mathematics are taught systematically from Reception onwards and pupils are given regular opportunities to develop the necessary skills.

Children are encouraged to reason and articulate their thinking, to verbalise their strategies, which helps to clarify them, not only to their classmates, but also to themselves.

Learning standard formal methods as a procedure and without conceptual understanding, before children have a firm grasp of place value, the four main operations and the number system in general, can inhibit the development of mental strategies and sound mathematical thinking. Therefore, we have developed a policy which scaffolds learning to reach more formal compact methods such as column addition, subtraction by decomposition, long multiplication and long division. When the children are fully secure and have all the numerical building blocks in place to fully appreciate and support their mental calculations they will be able to utilise these more formal compact methods appropriately. For most children this will tend to be towards the end of Key Stage 2.

Parent partnership

Whilst we value the contribution from parents in their children's education we have written this policy in order to avoid confusion to both parents and children. Parents are a vital link in their child's education and we endeavour to work in partnership with them throughout their child's educational journey at Reedham Primary. It is therefore crucial that parents are well informed of the methods taught in school, in order to allow them to support their children in the most appropriate way. As teachers, we need to work proactively and offer support and advice throughout the academic year. There are many ways in which we can support parents within mathematics and more particularly, with calculations:

- When sending homework out, teachers will include examples of methods taught in school – so that parents are aware of the methods taught and can continue them at home.
- Aid memoires can be sent home for children to keep (related to the four calculations) – this could encourage parents to consistently support their children with them throughout the whole academic year (not just when they are covering the particular units in school)
- Make use of parents evening sessions – to discuss and model methods to parents.
- Offer 'Mathematics' session on calculation methods (would benefit from being in the Autumn Term to equip parents with the understanding and methods early on in the year)
- Give opportunities for parents to join lessons with their children.

Aims

The overall aim of this policy is that when children leave school they:

- Have a secure knowledge of number facts and a good understanding of the four operations;
- Are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- Make use of representation and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- Have an efficient, reliable, written (procedural) method of calculation for each operation that children can apply with confidence (through conceptual understanding) when undertaking calculations that they cannot do mentally;
- Use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

Mathematics at Reedham Primary and Nursery School is taught using a mastery approach, which has number at its heart. Children are taught that everyone can do maths; children are taught as a whole class, with minimal differentiation. Children are taught conceptual understanding through representation, variation and mathematical thinking and fluency. Children are encouraged to look at what is the same and what is different and to explore mistakes. We believe mistakes are proof you are learning. Lessons are coherent and are well-planned, with small steps; each lesson is focused on one key learning point. The basis for planning and curriculum content used is the *White Rose* Premium Resources schemes of learning. Teachers are supported by Angles Maths Hub and are led by the in-school Maths Hub representative. Within our curriculum, a large proportion of time is spent reinforcing number to build competency, fluency, reasoning and problem solving, which underpin mathematical learning in the National Curriculum.

We try to ensure students have the opportunity to stay together as they work through the schemes as a whole group. Critical thinking and exploration in groups is encouraged through open ended activities. We consistently provide plenty of opportunities to build fluency, reasoning and problem-solving elements into every lesson. The basis for teaching is the use of a Concrete-Pictorial-Abstract approach. These varied representations underpin the children's mathematical thinking and their understanding of a concept. When introduced to a new concept, children should have the opportunity to build competency by exploring these stages to embed understanding:

Concrete – children should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing, for example, tens frames, base 10 and place value counters.
Pictorial – alongside this, children should use pictorial representations. These visual representations can then be used to help reason and solve problems. We encourage children to make jottings and draw their thinking.
Abstract – both concrete and pictorial representations should support children's understanding of abstract methods – written methods are a form of the abstract.

We use a calculation policy to support progression through the four calculations and fractions. Teachers use non-negotiables, so that maths is delivered consistently across the school. Teachers follow 's' plans, which incorporate small steps and include questions to ask children to encourage mathematical thinking, Stem sentences to support reasoning and vocabulary that will be used in each concept.

EYFS

We provide the children with a variety of resources in our continuous provision and planned adult-led activites, which are continually updated throughout the year. They are available to be used independently or as a class. The resources allow the children to learn in the continuous provision, focusing on Number and Shape, Space and Measure. Our displays are updated throughout the year with the children. The displays can then be referred to as a resource to support the children with their work.

Key Stage 1

The principal focus of mathematics teaching in key stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources [for example, concrete objects and measuring tools].

At this stage, pupils should develop their ability to recognise, describe, draw, compare and sort different shapes and use the related vocabulary. Teaching should also involve using a range of measures to describe and compare different quantities such as length, mass, capacity/volume, time and money.

By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency.

Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at key stage 1.

Lower Key Stage 2

The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.

At this stage, pupils should develop their ability to solve a range of problems, including with simple fractions and decimal place value. Teaching should also ensure that pupils draw with increasing accuracy and develop mathematical reasoning so they can analyse shapes and their properties, and confidently describe the relationships between them. It should ensure that they can use measuring instruments with accuracy and make connections between measure and number.

By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.

Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.

Upper Key Stage 2

The principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio.

At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems. Teaching in geometry and measures should consolidate and extend knowledge developed in number. Teaching should also ensure that pupils classify shapes with increasingly complex geometric properties and that they learn the vocabulary they need to describe them.

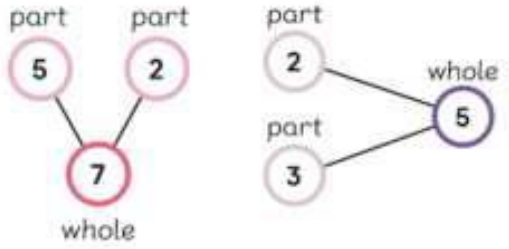
By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

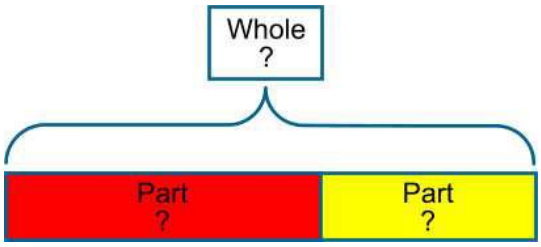
Pupils should read, spell and pronounce mathematical vocabulary correctly.

Number bonds

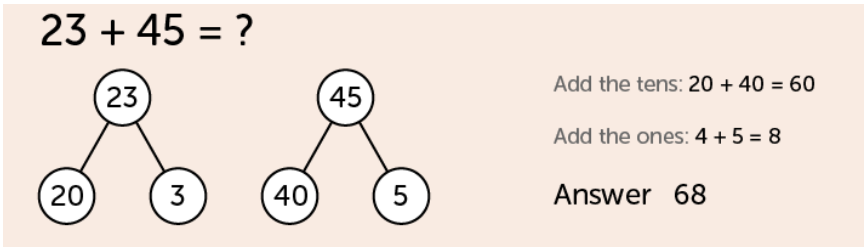
Number bonds refer to how numbers can be combined or added up. It is the 'part-part-whole' relationship of numbers. When talking about number bonds in maths mastery we are referring to how numbers join together and how they can be split up. A lot of emphasis is put into number bonds from EYFS so that children can build up their number sense prior to learning addition and subtraction. In the early stages students would be introduced to number bonds with concrete experiences, for example children could be given 6 linking cubes and guided to understand that 2 and 4 make 6, but that 1 and 5 also make 6.

The part-part-whole model can be shown in the following ways:





The mastery of number bonds is an important foundation that must be continually revisited throughout the child's school journey. It is required in subsequent mathematical learning and as a basis in the development of mental strategies. For example, children need to know that 7 + 3 = 10 to know that 7/10 + 3/10 = 10/10= 1 whole. A strong number sense allows students to decide what action to take when trying to solve problems in their head.



EYFS	Finding one more and one less with numbers to 10 Number bonds up to 5 e.g. 3 + ?, 5 – 2 =
Year 1	**Autumn** – Recap number bonds to 5, finding one more and one less with numbers to 10 **Spring** – Number bonds to 6,7,8, 9 e.g. 4+? = 9, 9 – 6 =? **Summer** – Number bonds to 10
Year 2	**Autumn** – Recap bonds up to 10 **Spring** – Number bonds to 11, 12, 13, 14, 15 **Summer** – Number bonds to 16, 17, 18, 19, 20
Year 3	**Autumn** – Recap number bonds up to 20 Number bonds to multiples of 10 up to 200 e.g. 20 + 60 = 80 **Spring** – Number bonds to 100 (e.g. 23 +?) Number bonds to 1000 (Multiples of 50 and 100 e.g. 350+650) **Summer** - Number bonds to 12 and 60 (to help with reading time) Number bonds up to 1 (tenths)
Year 4	**Autumn** - Recap number bonds to 100 and 1000 **Summer** – Recap number bonds to 12 and 60 (to help with reading time) Recap number bonds up to 1 (tenths) Number bonds with decimals and fractions to wholes
Year 5	Number bonds with percentages
Year 6	All number bonds (multiples of 10, 100 and 1000) Number bonds to whole

Addition

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and an efficient written method of calculation for addition which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method for addition of whole numbers by the end of Year 6.

Definition – *Addition is the process of combining 2 or more quantities.*

Early Learning

Students practise counting in 1s.

- They count chorally up to any given number;
- They count how many children would like school dinners;
- They count given objects to see how many they have;
- They sing counting songs.

Students use given apparatus or their fingers to find 1 more.

Students are introduced to written numbers.

Students are introduced to the + symbol.

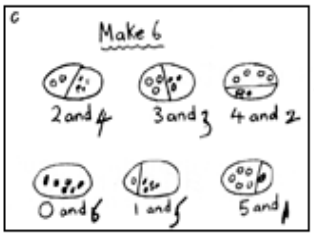
Begin to relate addition to combining two groups of objects.

- Make a record of pictures, words or symbols of addition activities already carried out.
- Construct number sentences to go with practical activities;
- Use games, songs and practical activities to begin using vocabulary;
- Solve simple word problems using their fingers.

Students will begin to double given equipment.
Students use their knowledge of the number system to count along a number line.

• Children use real life objects and apparatus to explore the different models of addition. 1. Augmentation in which two groups are combined: There are 3 footballs in the red basket, 2 footballs in the blue basket. How many footballs are there altogether? 2. Aggregation in which one group is added to: Sam has 3 marbles. Harry gives Sam 1 more marble. How many marbles does Sam have? • • Children are encouraged to develop a mental picture of	**Stage 1: Concrete**    **Stage 2: Pictorial**

the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc



- Children are taught to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equals sign so that the sign is not just interpreted as 'the answer'.

$2 = 1 + 1$
$2 + 3 = 4 + 1$
$3 = 3$
$2 + 2 + 2 = 4 + 2$
$4 + 2 = 2 + 2 + 2$

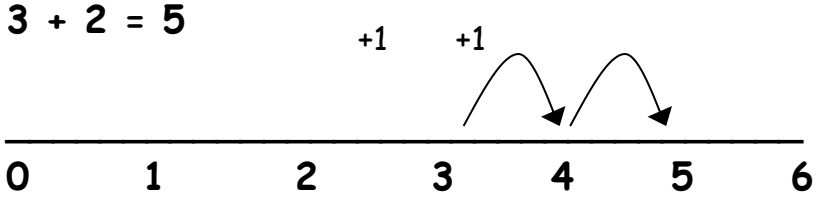
- Missing numbers are placed in different positions to challenge mathematical thinking.

$3 + 4 = \square$ $\quad\quad$ $\square = 3 + 4$
$3 + \square = 7$ $\quad\quad$ $7 = \square + 4$
$\square + 4 = 7$ $\quad\quad$ $7 = 3 + \square$
$\square + \nabla = 7$ $\quad\quad$ $7 = \square + \square$

- They use number lines and practical resources to support calculation and teachers demonstrate the use of the number line.

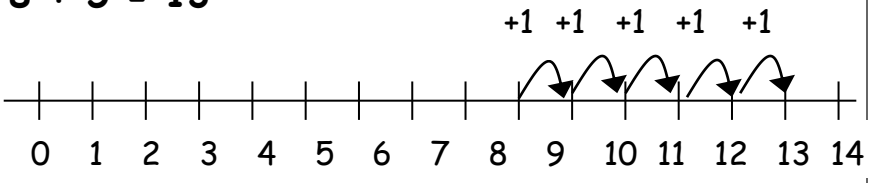
Stage 3: Number Line

3 + 2 = 5

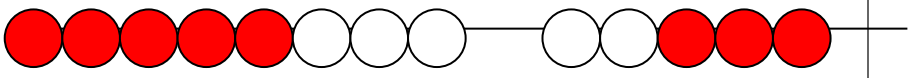


- Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

8 + 5 = 13



- Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.



- The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and ones separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.
- The empty number line helps to record the steps on the way to calculating the total.

Stage 4: Empty number line

Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.

8 + 7 = 15

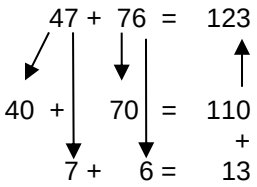
48 + 36 = 84

or:

- The next stage is to record mental methods using partitioning. Add the tens and then the ones to form partial sums and then add these partial sums.
- Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.

Stage 5: Partitioning

Informally record partitioning as follows:

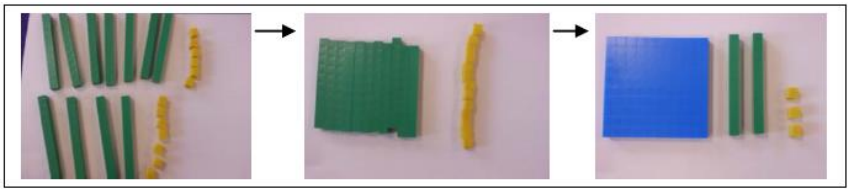


Progressing towards more formal recording:

47 + 76 = 40 + 70 + 7 + 6 = 110 + 13 = 123

Partitioned numbers are then written under one another:

$$\begin{array}{rcl} 47 & = & 40 + 7 \\ +\,76 & & 70 + 6 \\ \hline & & 110 + 13 \end{array} = 123$$



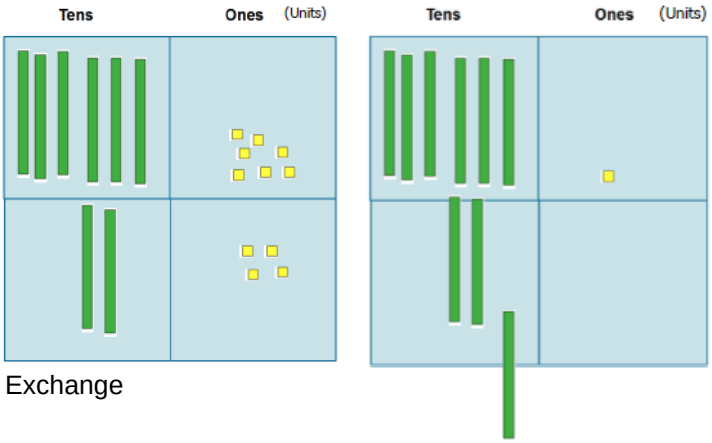
- Move on to a layout showing the addition of the tens to the tens and the ones to the ones separately. To find the partial sums either the tens or the ones can be added first, and the total of the partial sums can be found by adding them in any order. As children gain confidence, ask them to start by adding the ones digits first always.
- The addition of the tens in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing

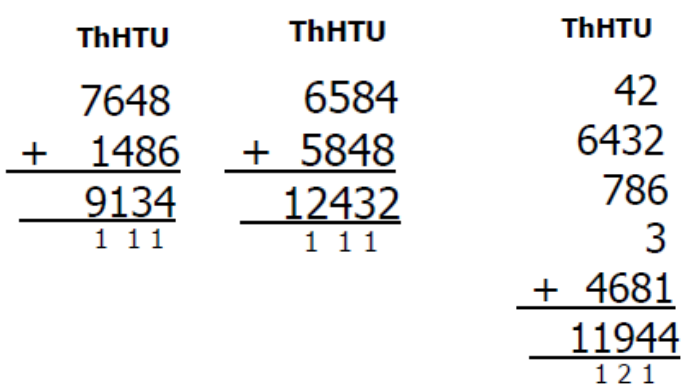
Stage 6: Expanded method in columns

Write the numbers in columns.

Adding the tens first:

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 110 \\ 13 \\ \hline 123 \end{array}$$

the link to the related fact 'four plus seven equals eleven'. • The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.	Adding the ones first: 47 + 76 13 110 123 Children are encouraged to discuss how adding the ones first gives the same answer as adding the tens first. Their procedural understanding will be refined over time to adding the ones digits first consistently.
• In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'. • Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits. • Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable. • Children should extend their column addition method with any number of digits	**Stage 7: Column Method** **(Preferred Method for Most Key Stage 2 Children)** 47 + 76 = 123 (11) 258 + 87 = 345 (11) 366 + 458 = 824 (11)  Exchange HTU 783 + 42 825 1 HTU 367 + 85 452 1 1

Subtraction

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for subtraction which they know they can rely on when mental methods are not appropriate.

At Reedham Primary & Nursery School, as a result of guidance from NCETM resources, Norfolk MathsHub and national researchers' children will be taught to use the empty number line approach to support and develop their ability to subtract. Level 5 pupils may move onto using an expanded written method of subtraction (decomposition) to improve the speed of their calculations if they can consistently show that they can solve subtraction questions successfully using this method.

Definition – *Subtraction is the inverse of addition. It can be defined as the process of taking away one number or amount from another, or as the act of finding the difference between two numbers or amounts.*

Early Learning

Students use given concrete apparatus or their fingers to find 1 less.
Students are introduced to written numbers.
Students are introduced to the - symbol.
Begin to relate subtraction to compare two groups of objects and find the difference.

- Make a record of pictures, words or symbols of subtraction activities.
- Construct number sentences to go with practical activities;
- Use games, songs and practical activities to begin using vocabulary;
- Solve simple word problems using their fingers.

Students will begin to halve given equipment.

Students use their knowledge of the number system to count forwards and backwards along a number line.

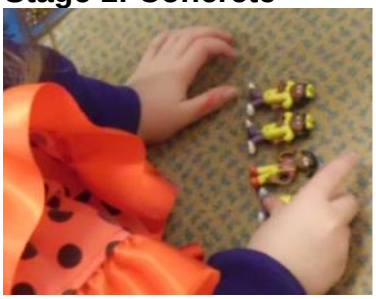
- Children use real life objects and apparatus to explore the different models of subtractions.
  1. Removing items from a set.

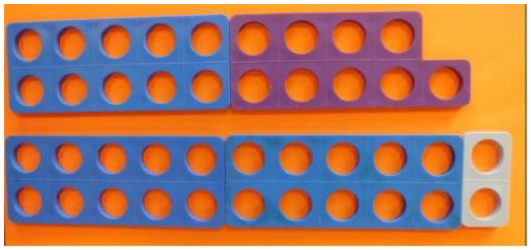
  2. Comparing two sets: (comparison or difference)

- Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.

- Children are shown representations of quantities in both scattered and ordered structures.

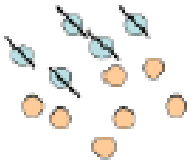
Stage 1: Concrete





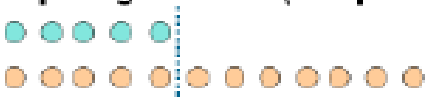
Stage 2: Pictorial

Removing items from a set (reduction or take-away)



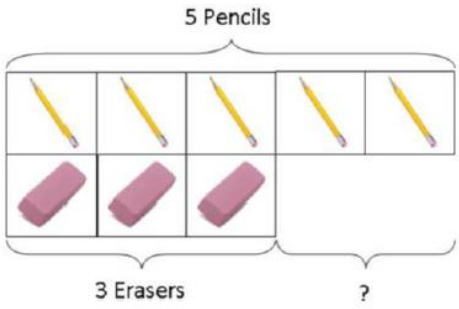
12 - 5 = 7

Comparing two sets (comparison or difference)



- Children are encouraged to use the Singapore bar model to order their thinking.

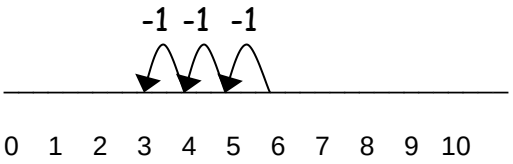
Peter has 5 pencils and 3 erasers, how many more pencils than erasers does he have?



- They use number lines and practical resources to support calculation. Teachers demonstrate the use of the number line.

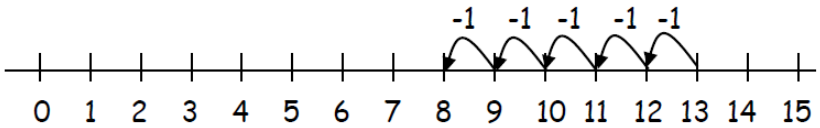
Stage 3: Number line (counting backwards)

6 – 3 = 3



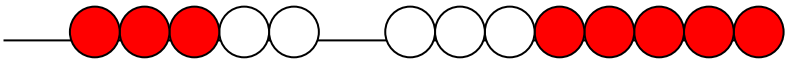
- Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

13 – 5 = 8



13 – 5 = 8

- Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.



- The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 - 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.

Stage 4: The empty number line

Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

15 - 7 = 8

- The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47.

74 - 27 = 47 worked by counting back:

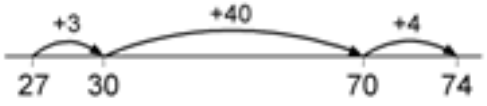
- With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 - 12, 86 - 77 or 43 - 28.

The steps may be recorded in a different order:

nental method of counting up from maller to the larger number can be ded using either number lines or ally in columns.

The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ? = 74 mentally.

The counting-up method (finding the difference, counting forwards)

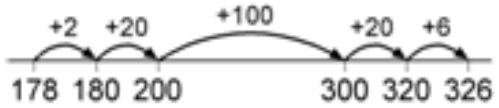


$$\begin{array}{rl} 74 & \\ -\ 27 & \\ \hline 3 & \rightarrow 30 \\ 40 & \rightarrow 70 \\ 4 & \rightarrow 74 \\ \hline 47 & \end{array}$$

Or:

$$\begin{array}{rl} 74 & \\ -\ 27 & \\ \hline 3 & \rightarrow 30 \\ 44 & \rightarrow 74 \\ \hline 47 & \end{array}$$

- With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ? = 200 and 200 + ? = 326 mentally.



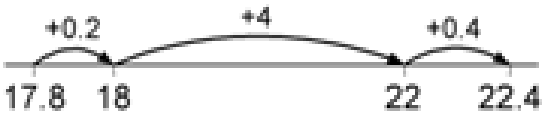
$$\begin{array}{rl} 326 & \\ -178 & \\ \hline 2 & \rightarrow 180 \\ 20 & \rightarrow 200 \\ 100 & \rightarrow 300 \\ 26 & \rightarrow 326 \\ \hline 148 & \end{array}$$

Or:

- The most compact form of recording is efficient if used appropriately.

326
−178
22 → 200
126 → 326
148

- The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed.



22.4
−17.8
0.2 → 18
4.0 → 22
0.4 → 22.4
4.6

- This counting-up method is a widely regarded best approach for most children as it supports their thinking and understanding of number.
- The children also have ownership of the workings out and can jump in amounts they feel comfortable with when solving a problem.

Or:

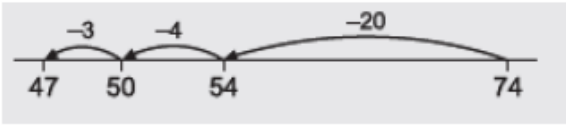
22.4
−17.8
0.2 → 18
4.4 → 22.4
4.6

- Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally.

Stage 5 – Partitioning

74 – 27 = 47

Children partition 27 into 20 and 7 and then subtracting from 74 the 20 and 7 in turn.
Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.



74 – 27 = 74 – 20 – 7 = 54 – 7 = 47

74 – 27 = 70 + 4 – 20 – 7 = 60 + 14 – 20 – 7 = 40 + 7 = 47

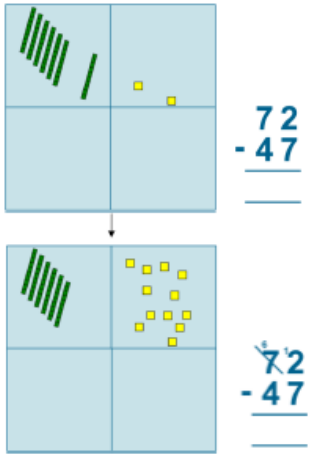
- The children should still be encouraged to use other methods where appropriate that would be more efficient when working out subtraction problems.
- This method relies on secure mental skills.
- Children write the calculation placing digits in their place value columns. Concrete materials are used to demonstrate when exchanging of tens takes place.
- Calculations become increasingly more complex

Stage 6 – Column method

Numbers are written under one another in the place value columns:

Example: 74 – 27

$$\begin{array}{r} {}^{6}\not{7}\,{}^{14}\not{4} \\ -\,2\,7 \\ \hline 4\,7 \end{array}$$



Concrete materials are used to demonstrate the exchanging of tens.

Example: 741 – 367

$$\begin{array}{r} {}^{6}\not{7}\,{}^{13}\not{4}\,{}^{11}\not{1} \\ -\,3\,6\,7 \\ \hline 3\,7\,4 \end{array}$$

Multiplication

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year4, two-digit by two-digit multiplication by the end of Year 5, and three-digit by two-digit multiplication by the end of Year 6.

To multiply successfully, children need to be able to:
recall all multiplication facts to 10 × 10;
partition number into multiples of one hundred, ten and one;
work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
add two or more single-digit numbers mentally;
add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
add combinations of whole numbers using the column method (see above).

Definition – *Multiplication is the product of two or more numbers or repeatedly adding a number or quantity. For example 4 multiplied by 5 (4 x 5) is 5 groups of 4 or 4 + 4 + 4 + 4 + 4. It is an inverse of division.*

All year 4 children participate in multiplication assessments, with this mind careful consideration is made to the progress they should make in the fluency of multiplication facts recall.

When?	Revise/practise	Teach
Yr1 Summer		0,1,10
Yr2 Autumn	10	2
Yr2 Spring	10, 2	5
Yr2 Summer	10, 2, 5	11 (Link to 10 groups + 1 group)
Yr3 Autumn	10, 2, 5, 11	3
Yr3 Spring	3	4 (Link to 2)
Yr3 Summer	3, 4	8 (Link to 2, 4)
Yr4 Autumn 1	4, 8	6 (Link to 3)
Yr4 Autumn 2	6	9 (Link to 3)
Yr4 Spring 1	All previous	7
Yr4 Spring 2	All previous	12 (Link to 3, 6, 4, 2)

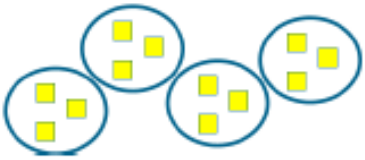
Early Learning

Students need opportunities to count groups of the same number of objects and add them together. They need a wide variety of experiences, engaging in songs, rhymes and real life contexts. Encourage students to draw pictures and to use equipment such as Numicon, beadstrings and cubes to show their representations. Students recognise that doubling and multiplying by 2 are the same.

- Children use real objects to experience the different models of multiplication.
- They understand that arrays, scaling, lots of and groups of are all multiplication.
- Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.
- Children will develop their understanding of multiplication and use jottings to support calculation by using repeated addition

Stage 1: Concrete

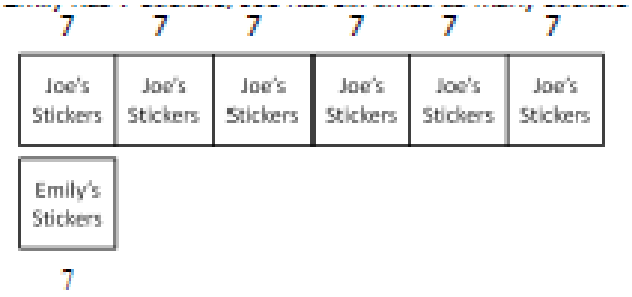
Stage 2 - Pictorial



- Children will be encouraged to use the Singapore bar model to order their thinking.

Singapore Bar Model

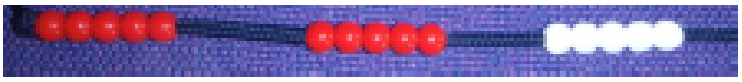
Emily has 7 stickers.
Joe has six times as many stickers as her.



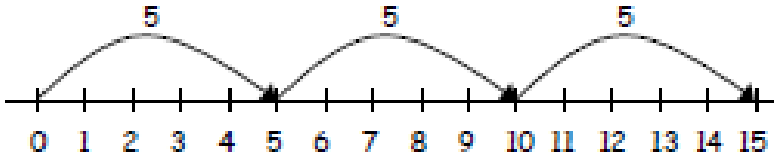
- Repeated addition can be shown easily on a number line and on a bead bar.

Stage 3 – The number line

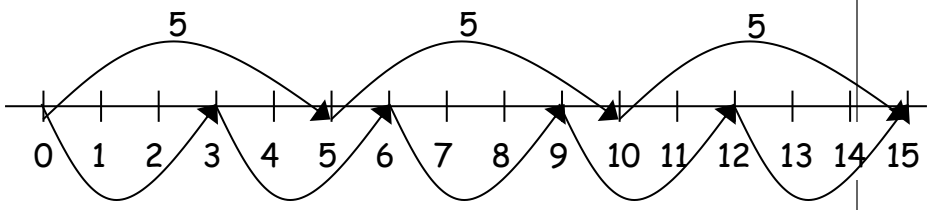
3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3



- Commutativity - Children should know that 3 x 5 has the same product as 5 x 3. This can also be shown on the number line.



3 X 5 is the same as 5 X 3

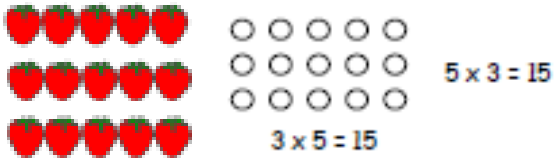


- Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.

Cuisinere rods demonstrate commutativity

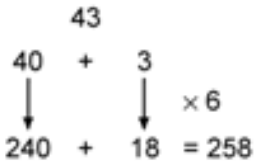
Stage 4 – Arrays

- Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.



Stage 5 – Partitioning

Informal recording in Year 4 might be:



Also record mental multiplication using partitioning:

14 X 3 =
10 X 3 = 30
4 X 3 = 12
= 42

43 X 6 =
40 X 6 = 240
3 X 6 = 18
258

<u>*Note:*</u> These methods are based on the distributive law. Children should be introduced to the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7:

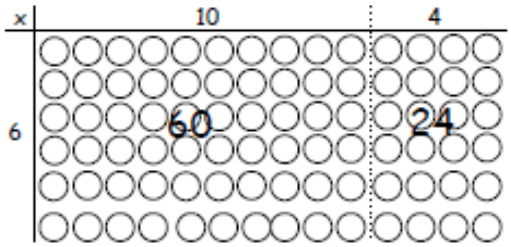
7 X 3 =

5 X 3 = 15
2 X 3 = 6

15 + 6 = 21

- As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps. This can be bridged by using representations with arrays.

Stage 6 – Grid method



(6 × 10) + (6 × 4)

60 + 24

84

- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

38 × 7 is approximately 40 X 7 = 280

X	7
30	210
8	56
	266

38 X 7 = 266

Children must have a secure understanding of place value to understand the grid method and demonstrate conceptual understanding

- Extend to TU × TU, asking children to **estimate** first.
- Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.

56 × 27 is approximately 60 × 30 = 1800.

X	20	7	
50	1000	350	1350
6	120	42	162
			1512

1

56 X 27 = 1512

- Extend to HTU × TU asking children to estimate first. Start with the grid method.
- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

286 × 29 is approximately 300 × 30 = 9000.

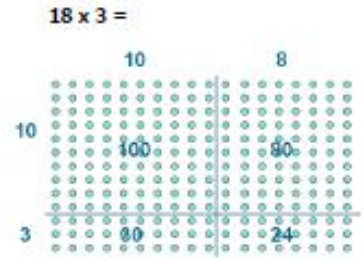
X	20	9	
200	4000	1800	5800
80	1600	720	2320
6	120	54	174
			8294

1

286 X 29 = 8294

Stage 7 - Expanded Short Multiplication Method

- The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.
- Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.
- The expanded method can be used with progressively harder examples moving onto two-digit number multiplied by a two-digit number and then onto three-digit number multiplied by a two-digit number.
- In the examples given, encourage the children to estimate the answer before working on the calculation. It is important that the children continue to look at the numbers carefully to support their thinking and understanding.



Arrays can demonstrate conceptual understanding. It is important that children use concrete materials and pictorial representations to support their thinking.

38 X 7 is approximately 40 X 7 = 280

$$\begin{array}{r} 30+8 \\ \times\ \ 7 \\ \hline 210 \\ 56 \\ \hline 266 \end{array} \qquad \begin{array}{l} 30\times7=210 \\ 8\times7=\ 56 \end{array} \qquad \begin{array}{r} 38 \\ \times\ \ 7 \\ \hline 210 \\ 56 \\ \hline 266 \end{array}$$

56 × 27 is approximately 60 × 30 = 1800.

$$\begin{array}{r} 56 \\ \times\ 27 \\ \hline 1000 \\ 120 \\ 350 \\ 42 \\ \hline 1512 \\ 1 \end{array} \qquad \begin{array}{r} 50\times20=1000 \\ 6\times20=\ 120 \\ 50\times7=\ 350 \\ 6\times7=\ 42 \end{array}$$

286 X 29 is approximately 300 X 30 = 9000.

$$\begin{array}{r} 286 \\ \times\ 29 \\ \hline 4000 \\ 1600 \\ 120 \\ 1800 \\ 720 \\ 54 \\ \hline 8294 \\ 1 \end{array} \qquad \begin{array}{r} 200\times20=4000 \\ 80\times20=1600 \\ 6\times20=\ 120 \\ 200\times\ 9=1800 \\ 80\times\ 9=\ 720 \\ 6\times\ 9=\ 54 \end{array}$$

• The recording is reduced further, with carry digits recorded below the line. • If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 5 (b).	**Stage 8 – Short Multiplication Method** 38 x 7 266 5 The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage. 286 x 29 2574 (9 x 286 = 2574) 7 5 5720 (20 x 286 = 5720) 1 1 8294 1 Children, who are already secure with the formal written method multiplication for TU x U and TU x TU should have little difficulty in using the same method for HTU x TU and ThHTU x TU

Division

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for division which they know they can rely on when mental methods are not appropriate.

To divide successfully in their heads, children need to be able to:

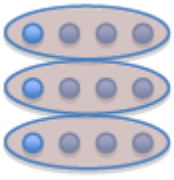
- understand and use the vocabulary of division - for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally - for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

To carry out written methods of division successfully, children also need to be able to:

- understand division as repeated subtraction;
- understand division can be worked out as repeated addition;
- estimate how many times one number divides into another - for example, how many sixes there are in 47, or how many 23s there are in 92;
- multiply a two-digit number by a single-digit number mentally;
- subtract numbers using the number line approach.

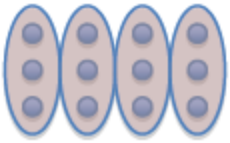
Definition - division is the inverse of multiplication and is the way of determining how many times one quantity is contained within another. Either sharing or grouping can divide a quantity.

Grouping a quantity



How many groups of 4 are there in 12?
12 ÷ 4

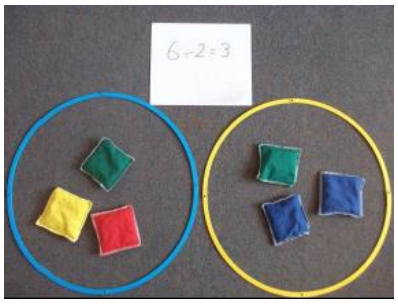
Sharing a quantity

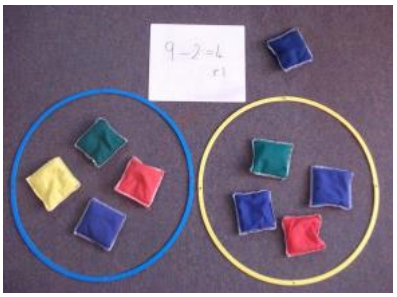


Sharing 12 objects between
12 ÷ 3

- Children will understand equal groups and share items out in play and problem solving. They will count concrete resources in 2s and 10s and later in 5s.

Stage 1 – Concrete



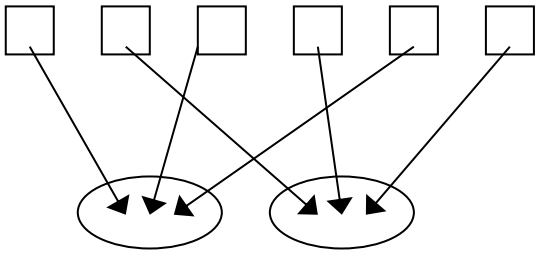


- Children will develop their understanding of division and use jottings to support calculation – linking to division as sharing equally

Stage 2 – Pictorial

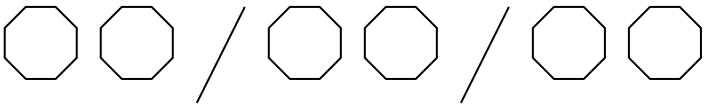
6 sweets shared between 2 people, how many do they each get?

- - or linking to Grouping

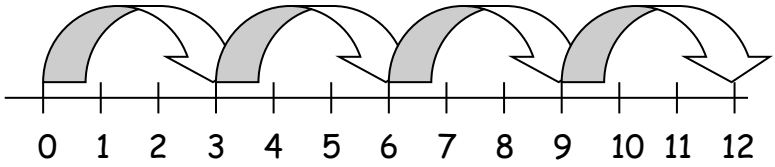


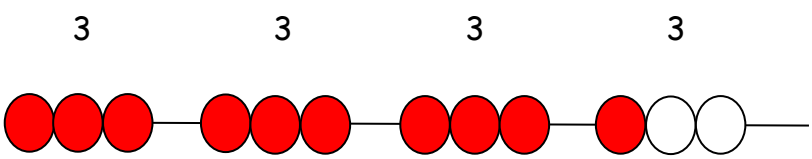
There are 6 sweets, how many people can have 2 sweets each?

- Then moving onto repeated addition (chunking) using a number line or bead bar



12 ÷ 3 = 4



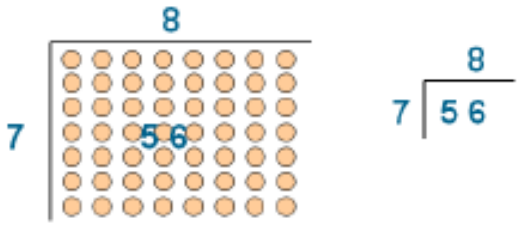


- Progressing onto using symbols to stand for unknown numbers to complete equations using inverse operations

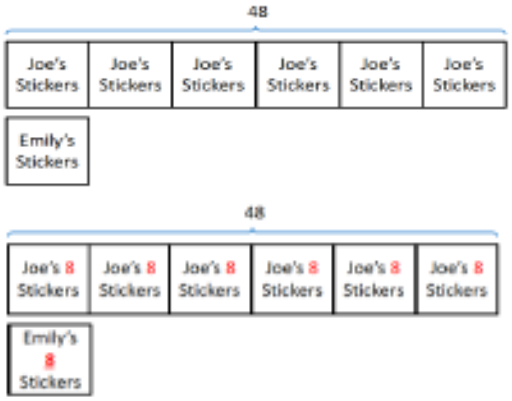
- Arrays can be used to show why division calculations are sometimes laid out like this. We are looking for the missing side of the array.

- The bar model is a useful representation to solve division problems.

□ ÷ 2 = 4 20 ÷ △ = 4 □ ÷ △ = 4



Joe has 6 times as many stickers as Emily.
Joe has 48 stickers.
How many stickers does Emily have?

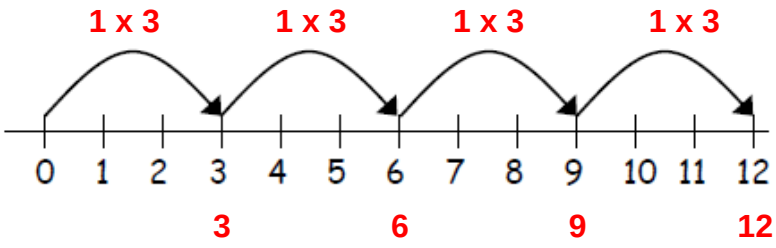


It is clear that Emily also has 8 stickers, 48 ÷ 6 = 8

- Children will use an empty number line to support their calculation.

- It is important that children begin to annotate their numberline when chunking (by addition) so that they begin to see that they are not only counting in 4's (repeated addition) but using their times table facts too. They will have a firmer basis to then develop 'chunking up' on a number line into year 4.

Stage 3 – The number line



- Children will progress onto an empty number line.

- Children should also move onto calculations involving remainders.

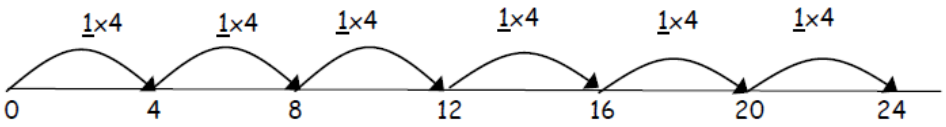
- This is built upon from the chunking number line work from Years 2 and 3 – children will progress by chunking in bigger steps and using their times table facts to support this calculation.

- Children use related multiplication facts to support their calculations as jottings.

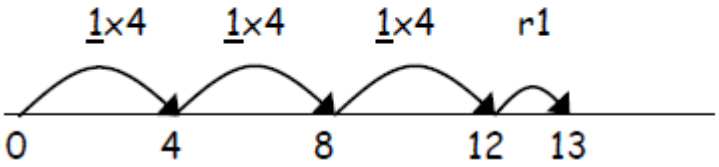
- Children should use symbols to stand for unknown numbers to complete equations using inverse operations.

Stage 4 – The empty number line

24 ÷ 4 = 6



13 ÷ 4 = 3 r 1



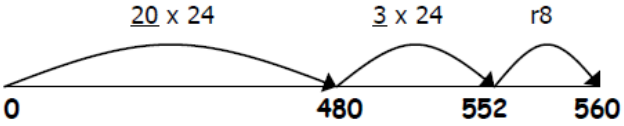
Children see that division is the inverse of multiplication.

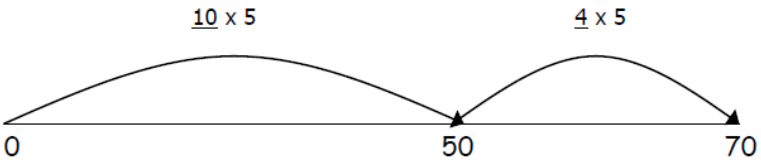
70 ÷ 5 = 14
How many 5s in 70?
10 lots of 5 and 4 lots of 5.
How many altogether?

1 x 24 = 24
2 x 24 = 48
5 x 24 = 120
10 x 24 = 240
20 x 24 = 480
50 x 24 = 1200
100 x 24 = 2400

HTU ÷ TU

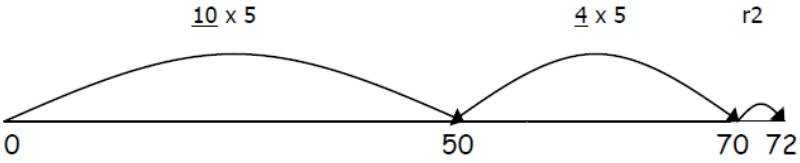
560 ÷ 24 = 23 r 8





Division with remainders:

72 ÷ 5 = 14 r 2



26 ÷ 2 = □ 24 ÷ △ = 12 □ ÷ 10 = 8

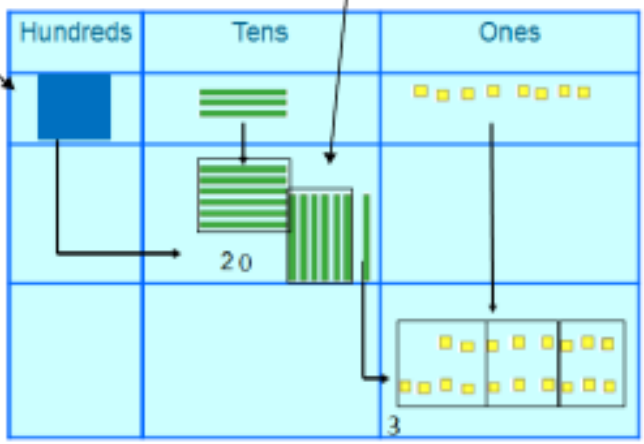
- Children use base ten representations and multiplications facts as jottings to support calculations. They use 'bus stop' method to record their calculations.

Stage 5 – Short division

Use Base 10 representations to show what is happening: $6\overline{)138}$

1. Although we know that we can fit sixes into 100, this 100 square cannot be physically broken into groups of 6 so we exchange it for 10 tens.

2. We now have 13 tens. We can break them into 2 groups of 6 tens with 1 ten left over, which we exchange for ones.
The 2 groups of 6 tens represent 20 lots of 6. We write the 2 in the tens column above the line in our calculation.



3. We now have 18 ones. We can group these into three lots of six. So we write 3 in the units column above the line in our calculation.
Altogether there are 23 sixes.

$$\begin{array}{r} 23 \\ 6\overline{)138} \end{array}$$

- The next step is to tackle HTU ÷ TU, which for most children will be at the end of Year 6.
- The layout on the right, which links to chunking, is in essence the 'long division' method.
- Recording the build-up to the quotient on the left of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient.
- Conventionally the 20, or 2 tens, and the 3 ones forming the answer are recorded above the line, as in the second recording.
- Children can progress to decimal calculations once secure.

Stage 6 – Long division

How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate.

As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560.

$$\begin{array}{rrl} 24\overline{)560} & & \\ 20 - & \underline{480} & 24\times20 \\ & 80 & \\ 3 & \underline{72} & 24\times3 \\ & 8 & \end{array}$$

Answer: 23 R 8

In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below.

$$\begin{array}{r} 23 \\ 24\overline{)560} \\ -\underline{480} \\ 80 \\ -\underline{72} \\ 8 \end{array}$$

Answer: 23 R 8

$$\begin{array}{r} 22.5 \\ 6\overline{)^{1}1^{1}3^{1}5.^{3}0} \end{array}$$

Fractions

Fractions form a very large portion of the knowledge children must acquire in the maths curriculum, particularly in KS2. It is important for us to teach this in a sequential order to support this knowledge and understanding.

Year 1	• Name the fractions 'one-half', 'one-quarter' and 'one-third' in relation to a fraction of a length, shape or set of objects.  • Read and write the fraction notation $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ and relate this to a fraction of a length, shape or set of objects.
Year 2	• Find half of numbers. • Find $\frac{1}{3}$ or $\frac{1}{4}$ of a number. Find $\frac{2}{4}$ and $\frac{3}{4}$ of an object, shape, set of objects, length or quantity; recognise equivalence of $\frac{2}{4}$ and $\frac{1}{2}$.
Year 3	• Identify parts and wholes of areas, lengths and sets. Identify equal and unequal parts; make judgements about the relative size of a part to a whole. Find the whole when the size of a part and number of equal parts is known. Use the same shape to explore parts and wholes. • Learn to name and write unit fractions. Recognise and show unit fractions of areas, lengths and quantities. Relate numerators and denominators to parts and wholes; explore how the greater the denominators, the smaller the unit fraction. • Explore how to add and subtract fractions within one whole where the denominators are the same. Apply prior knowledge of the inverse relationship of addition and subtraction with whole numbers, to fractions.
Year 4	• Meet mixed numbers and improper fractions, and learn to convert between them; compare, order and place them on a number line. Extend addition and subtraction from within a whole to numbers greater than one whole. • Consider multiplication of whole numbers and proper fractions as both repeated addition and scaling. Understand that multiplication of a whole number by a proper fraction results in a smaller number
Year 5	• Discover how equivalent fractions have the same proportional relationship between the numerator and denominator, and therefore have the same numerical value. Convert between equivalent fractions and simplify fractions. • Learn to add and subtract fractions with different denominators by first finding a common denominator. Compare fractions using a range of methods, including converting to a common denominator.
Year 6	• Explore how to multiply two fractions. Learn how to divide a fraction by a whole number by first converting to an equivalent multiplication. Review how multiplying by a proper fraction makes a number smaller. • Make connections between fractions and previous work on decimals. Learn common fraction and decimal equivalences. Understand that percentages tell us about the proportion being considered. Find percentages of quantities.

Appendix 1 – Key Vocabulary

Addition	Subtraction
Add Addition Plus More than Altogether Sum Total Increase Count up	Count back Count up Decrease Fewer Find the difference Difference between Subtract How many left? How many more? Less Less than Minus Reduce Take away Take away from
Multiplication	**Division**
Multiply Multiplies Times Groups of Lots of Repeated addition Arrays Multiplied by Product	Grouping Sharing Divisor Remainder Repeated subtraction Dividend Share equally Divided by

We all know that mathematical terms can be difficult to understand, so here is a link to an online Maths Dictionary that can help explain any mathematical terms.

https://www.mathsisfun.com/definitions/letter-c.html

Appendix 2 – Using Representations in Maths

Representations are a useful tool in mathematics to support learners in developing conceptual understanding through: communicating, reasoning, solving problems, making connections between ideas and learning new concepts. (Principles and Standards of School Mathematics (NCTM, 2000) on https://learner.org/courses)

> "How we understand something is we see how it is related or connected to other things we know.'
>
> J.Hiebert, Signposts for Teaching Mathematics through Problem Solving 2003

With conceptual understanding we:

- Know more than just facts;
- Know why a mathematical idea is important;
- Learn new ideas by connecting them to the ones we already know;
- Can remember or retain ideas.

Often older and higher-attaining students view practical resources as a tool for those who find mathematics difficult – we should actively challenge this perception and ensure learners of all ages and stages have the opportunity to deepen their levels of understanding through the use of representations; allowing them to become mathematicians and not just a follower of mathematical processes. Practical approaches to work can often mean the knowledge and understanding is more likely to be retained after the session has ended.
The following image shows some of the wide range of materials that teachers can draw upon when planning to use representations in their mathematics lessons.

Appendix 3 – Use of Technology

Technology should play an important role in developing the student's practical calculation skilss. Technology can be used to promote the thinking and reasoning skills needed for problem solving within mathematics as well as increasing the student's understanding of arithmetic operations and numerical relationships. The use of technology should be linked to the appropriate age and ability of the stage the student is working at. It should be carefully planned to advance learning within the classroom and it should not replace the need for students to develop efficient and accurate methods for both mental and written calculations as well as performing sensible estimations. Technology resources that may be used to support the teaching and learning of calculations could include:-

- Calculators
- Computers
- Tablets
- Interactive Whiteboards
- Promethean Slates
- Bee-bots

Suggested Apps:

Addition & Multiplication (Unripe Grape)

Matholku Lite

Maths Party

Math Academy Free

Suggested websites:
www.stemnet.org.uk
www.mathsframe.co.uk
www.ncetm.org.uk
nrich.maths.org
youcubed.stanford.edu

- We also use Maths Whizz as an intervention to support key skills and topics.
- We encourage children to practise their calculation skills using Numbots and Times Table Rock Stars.
- We use ‘mad minutes’ where children complete multiplication facts in a minute – they set themselves personal challenges.
- We use Weekly Basics in Upper Key Stage 2 to practise mental strategies and calculations.